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METEOSAT. PCM FRAME TRANSMISSION FORMAT DEFINITION

No Author

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METEOSAT

PCM FRAME TRANSMISSION FORMAT DEFINITION

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ABSTRACT. PCM transmission of satellite photographs is analyzed. Several possible coding schemes are evaluated. A single synchronization code per line with an alternating and nonalternating code is considered.

I. INTRODUCTION /1*

Format definition for PCM transmission depends on a number of criteria:

- probability of true detection, which is the probability of recognizing the synchronization code,
- probability of bit error, which depends on transmission quality,
- bit shift introduced by the primary synchronizer,
- equipment feasibility and cost.

1. Probability of True Detection

A frame contains 2,000 lines and is transmitted in one-half hour. The maximum number of lines per day is, therefore, the following:

* Numbers in the margin indicate the pagination of the original foreign text.

$$2000 \times 48 = 96.000 \simeq IO^5$$

It may be assumed (to a first approximation) that detection of the synchronization code for each line is independent of the others. In this case, if Pv is taken as the probability that one line is correct, then the probability that all lines are correct can be expressed as $Pc = Pv^{10^5}$.

$$\text{If} \quad Pf = I - Pv \qquad Pc = (I - Pf)^{IO^5} \simeq I - IO^5\,Pf \quad \begin{matrix}\text{since} \\ Pf \ll I\end{matrix}$$

It is desired that there be no more than one false line per day. This can be expressed by:

$$\frac{Pc}{I-Pc} \geqslant IO^5$$

With Pc as the probability that all lines are correct, and with 1-Pc as the probability that there is at least one false line: /2

$$\frac{I - IO^5\,Pf}{IO^5\,Pf} \geqslant IO^5$$

$$I - IO^5\,Pf \geqslant IO^{IO}\,Pf$$

$$\boxed{Pf \leqslant IO^{-IO}}$$

From the frame point of view, if Ps is used to designate the probability that all lines in a frame will be correct, then:

$$Ps = Pv^{2000} = (I-Pf)^{2000} \doteq I - 2x\,IO^3\,IO^{-IO}$$

$$Ps = I - 2 \times IO^{-7}$$

It is condition $Pf \leqslant IO^{-IO}$ which will determine length of the synchronization word.

2. Probability of Bit Error

The information contained in a PCM transmission is a physical quantity coded in digital form. If the coding is to have meaning, the probability of bit error due to transmission quality must introduce an error lower than the quantization error.

Since nine bits are used to code infra-red, this corresponds to the following signal-to-noise ratio:

$$\frac{S}{N} = 64 \text{ dB}$$

and to the ratio:

$$\frac{E}{No} = 11 \text{ dB (see paper 487/DB/ES/E by A. Pouzet)}$$

For a code NRZ-L or ØØ-L , bit error probability is related to the ratio $\frac{E}{No}$ by:

$$p = \frac{I}{2} \left[I - \mathrm{Erf} \sqrt{\frac{E}{No}} \right]$$

Which gives: $\boxed{p = 10^{-6}}$

One can expect, then, a bit error probability of 10^{-6}. In fact, we will determine the code for a bit error probability as great as 10^{-2}.

3. Bit Shifts /3

Bit shifts are caused by synchronization defects in the primary synchronizer. They manifest themselves, from time to time, as a skipped or doubled bit.

The probability of bit shift diminishes as the signal-to-noise ratio increases, but this depends on the individual equipment and can only be properly evaluated through long duration tests. One may simply conclude that with a E/No ratio of 11dB, this phenomenon will be practically nonexistent.

If that is verified, a single synchronization word at the beginning of each line would be sufficient. Otherwise, it is preferable to break up the line in such a way that repetitive rephasing minimizes the amount of information lost at each bit shift.

4. Equipment Feasibility and Cost

The most economical solution would be to use off-the-shelf equipment. Commercial secondary synchronizers are limited to a few thousand bits per cycle and to 33-bits for the synchronization word.

Breaking up the line would allow shortening the cycles, but the synchronization word (see Paragraph IV) would always have a critical length.

For on-board equipment, coder storage depends on the type of code used. A code with N bits will require a storage with N positions, while a PN code (with length $N = 2^n - 1$) can be achieved with n flip-flops in the circulating memory.

Several solutions satisfy these various criteria:

- a single synchronization word can be used at the beginning of each line; or,
- the line is broken up. In this case, a reference mark for line beginning must be used in addition to the synchronization word.

II. A SINGLE SYNCHRONIZATION CODE PER LINE (NONALTERNATING CODE) /4

The first possible solution is the use of a synchronization code placed at the beginning of each line, which means that a cycle is made up of one complete frame line. In addition, the code chosen repeats in identical form from one line to another.

1. Code Length

Code length is determined by the probability for correct synchronization which has been selected.

Variable threshold search provides better results than fixed threshold (see Appendix I) so it is the one we have used hereafter.

Given: N Length in bits of the synchronization word
M Total number of bits in the cycle
P Probability of bit error
E Number of code errors
e Number of errors at any position in the cycle.

Probability Pv of detecting the synchronization code is given by the following (see Appendix I):

$$Pv = \sum_{E=o}^{N/4} C_N^E \; p^E \; (I-p)^{N-E} \left[I - \left(\frac{I}{2}\right)^N \sum_{e=o}^{E} C_N^e \right]^{M-I}$$

In the case of METEOSAT, M is on the order of 70,000 bits.

The various values of Pv as a function of N and P are shown on the curve in Figure 1.

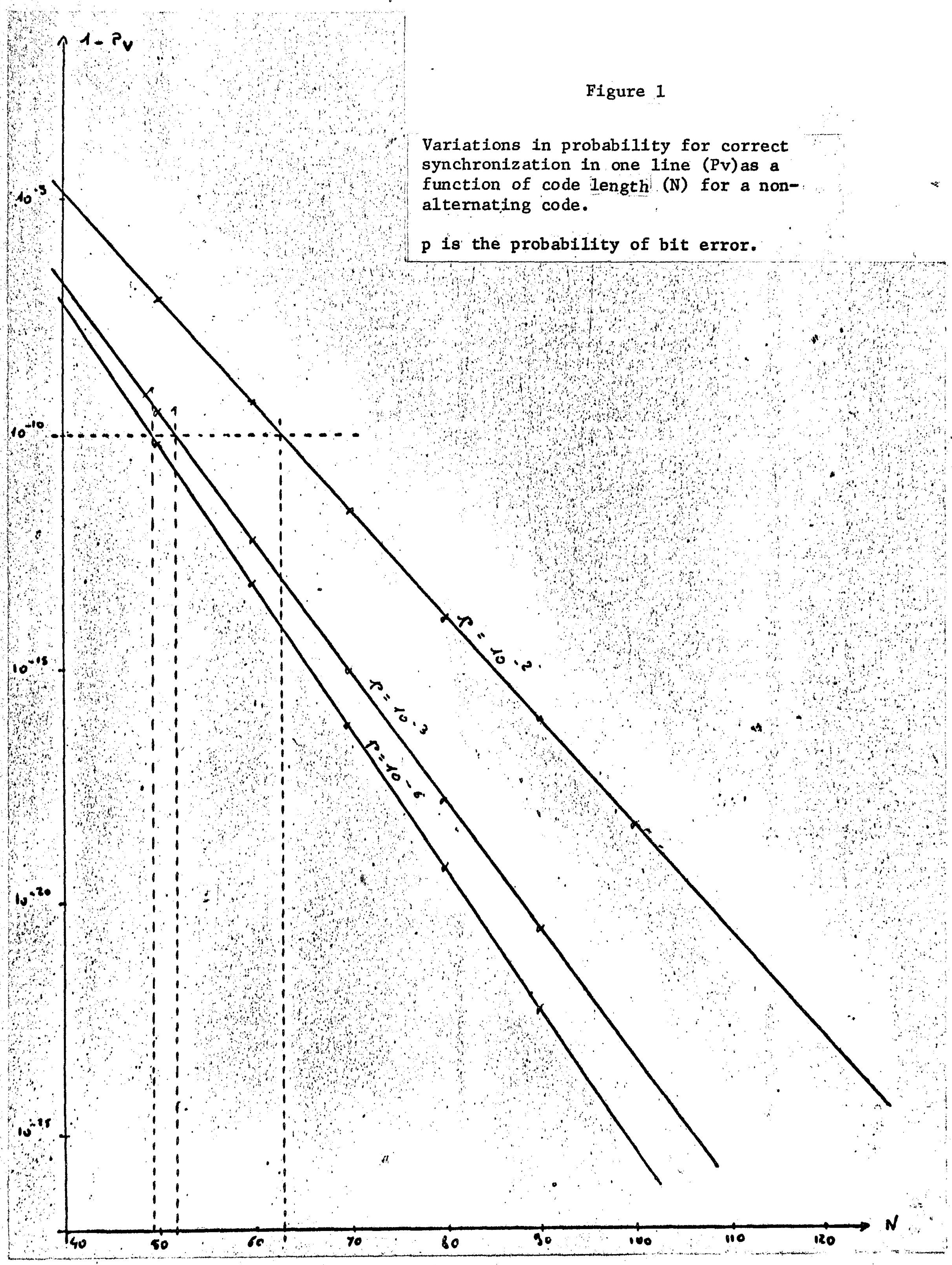


Figure 1

Variations in probability for correct synchronization in one line (Pv) as a function of code length (N) for a non-alternating code.

p is the probability of bit error.

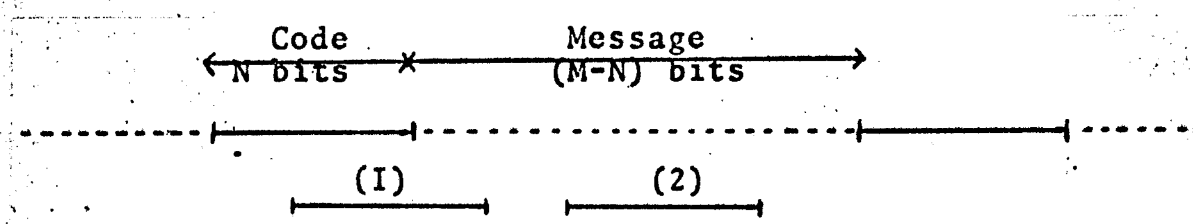
A length of 63 bits is required to obtain a probability Pv of $1 - 10^{10}$ for a p of 10^{-2},

$$\boxed{N = 63 \text{ bits} \longrightarrow Pv = I - 9{,}6I5 \quad IO^{-II}}$$

2. Composition of the Code /5.

The composition of the code must be such that the hypothesis outlined earlier (namely, that all bits are random outside of the exact code position) is not violated.

Among the M-1 positions other than the code, two cases can be noted:



For a position such as position (1), the correlator contains code bits (nonrandom bits) and message bits (random bits) at the same time, while in a position such as (2) it contains only random bits.

The hypothesis made earlier (random bits for all positions outside the code) is justified if calculation of Pv taking code composition into account provides as good a value as when it was calculated earlier (Paragraph II.1).

A number of studies have been completed on code compositions [References 3, 4, 5], but they were done for fixed threshold search for code lengths not exceeding 30 bits. In addition, the assumptions are p = 0.1 and E = 0 or 2 (E is the error threshold for the fixed threshold method).

These studies are not directly applicable to METEOSAT; therefore, they must be redone for an assumed variable threshold.

Positions such as position (1) defined above are 2N - 2 in number, each of them characterized by its overlay m (see Appendix 3). Each value of m corresponds to two symmetrical positions (an overlay to the right and an overlay to the left) which have the same characteristics.

If one uses Hm(E) to designate the probability that the number of errors (for an overlay m) will be equal to or lower than E, then probability Pv is expressed as follows:

$$Pv = \sum_{E=o}^{N/4} C_N^E \, p^E \, (I-p)^{N-E} \left[\prod_{m=I}^{N-I} (I-Hm(E)) \right]^2 \left[I - \left(\tfrac{I}{2}\right)^N \sum_{e=o}^{E} C_N^e \right]^{M-2N+I} \qquad /6$$

If c is the number of mismatches in the truncated correlation, Hm(E) is expressed by (see Appendix 3):

$$Hm(E) = \left(\tfrac{I}{2}\right)^b \sum_{i=Max(o,c-E)}^{c} C_c^i \, p^i (I-p)^{c-i} \sum_{j=o}^{Min(m-c,E-c+i)} C_{m-c}^j \, p^j (I-p)^{m-c-j} \sum_{\beta=o}^{Min(E-c+i-j,b)} C_b^{\beta}$$

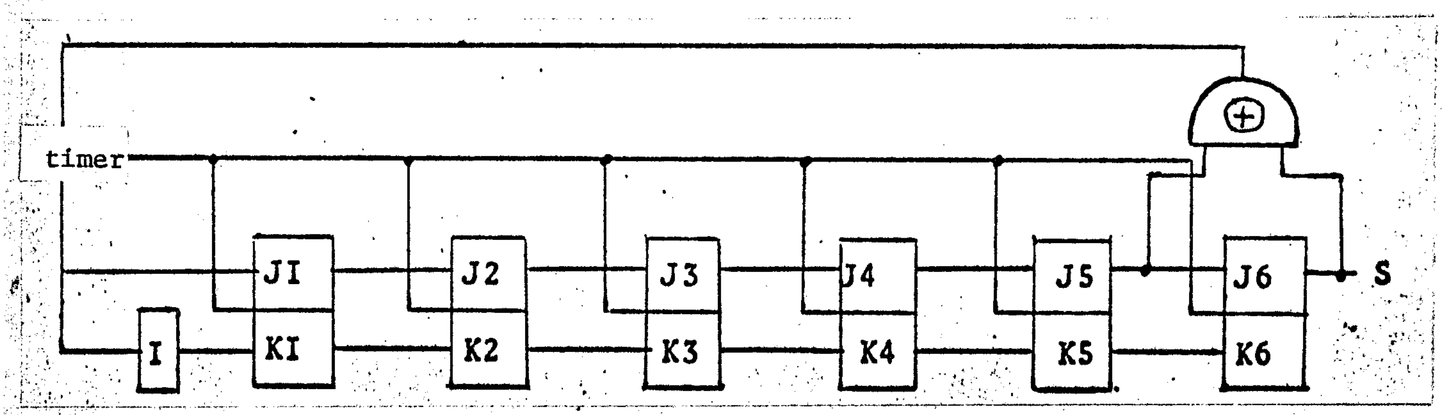
with b = N-m

A code is good if Pv is higher than the value computed with the first hypothesis (Paragraph II.1); that is, $1-9.615\ 10^{-11}$.

A search for the best code is not contemplated since (for 63 bits) there would be 2^{63} different codes to examine. Even dividing by four to take into account mirror, complementary, and complementary-mirror codes (which have the same characteristics), the computation time would be disproportionate with respect to the results which might be expected.

It is enough, then, to find a good code, even if it isn't the best.

Since $63 = 2^6-1$, the 63-bit code is advantageous since it can be formulated by a PN code with six flip-flops. It is known that a PN code has good correlation properties, even if it is not necessarily optimum. It can be produced with the following schematic:



⊕ is a module 2 (or exclusive) addition

I is an inverser (complementary)

A code S (the output) is then obtained: /7

IIIIIIOOOOOIOOOOIIOOOIOIOOIIIOIOOOIIOOIOOIOIIOIIIOIIOOIIOIOIO

with its 63 rotary permutations.

The 63 codes thus obtained give Pv values which are very close one to the other and better than the value obtained using the hypothesis that all bits are random.

1-Pv varies between 9,61111 10^{-11} and 9,59844 10^{-11} in place of 9,61541 10^{-11}

For other codes, in way of contrast, the probabilities for true detection are very low. Values of 1-Pv become, for example:

0,70722I	IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
0,283II	IIOIIOIIOIIOIIOIIOIIOIIOIIOIIOIIOIIOIIOIIO
0,46067	IOIOIOIOIOIOIOIOIOIOIOIOIOIOIOIOIOIOIOIOIOIO
7,3766 IO^{-5}	OOOOIIIOIIOIOOOIIIOIIOIOOOIIIOIIOIOOOIIIOI

IIIIIIIIIIIIIIIIIIIII
IIOIIOIIOIIOIIOIIOIIO
IOIOIOIOIOIOIOIOIOIOI
IOIOOOIIIOIIOIOOOIIOI

The last code is composed of:

an 0
followed by an 11-bit Barker code, repeated five times
followed by a 7-bit Barker code.

The earlier random code is, therefore, perfectly suitable.

Another criterion will be required to make a choice among the 63 permutations -- the best probability to remain locked on the code taking into account the width of the window. This problem has not yet been examined.

III. A SINGLE SYNCHRONIZATION CODE PER LINE (ALTERNATING CODE) /8

A telemetry cycle is made up of a METEOSAT frame line which has a fixed character due to the fact that information varies little from one line to another. As a result, for a given position in the line there is a chance of finding the identical bit configuration in several consecutive lines. If the secondary synchronizer locks on such a position, there is a chance it will lock there.

The use of an alternating code reduces this problem -- from each line to the next, the synchronization code is the complementary code of the code used for the preceding line.

For the same probability of false detection as with a nonalternating code, this method reduces probability of locking on a wrong position.

1. Code Length

Code length is determined, as in the preceding case, by selected probability of correct synchronization.

The synchronization code at the beginning of each cycle is alternately S or $\bar{S}$. In search phase, consequently, the code expected can be one or the other.

It can be deduced therefore that Pv (true detection probability) is:

$$P_v = 1/2 \text{ Prob (lock on } S \text{ if } S \text{ is the code)} + 1/2 \text{ Prob (locks on } \bar{S} \text{ if } \bar{S} \text{ is the code)}$$
$$= 1/2\ (P(S) + P(\bar{S}))$$

The secondary synchronizer always functions with a single correlator for which the reference code is S. Consequently, the number of errors obtained at the output is the number of mismatches between S and that part of the PCM carrier contained in the correlator. N-e is the number of mismatches with $\bar{S}$.

Let us look at a variable threshold operation and begin by computing P(S).

If E is the number of errors detected on the code, and if S is the expected configuration, then detection will only be correct if:

$$N - E > E$$ /9

that is:

$$E < \frac{N}{2}$$

For the same reasons as in Paragraph II.1, E is limited to N/4. The probability P(S) of a true detection on S is the probability that the number of mismatches with S or $\bar{S}$ found on preceding positions will be higher than the number of mismatches with S found in this position.

That is

$$e > E$$

$$N-e > E$$

The probability sought is expressed, then , by:

$$P(S) = \sum_{E=o}^{N/4} C_N^E \, p^E \, (I-p)^{N-E} \left[\sum_{e=E+I}^{N-E-I} \left(\frac{I}{2}\right)^N C_N^e \right]^{M-I}$$

Analogous reasoning for $\bar{S}$ leads to $P(\bar{S}) = P(S)$.

From which:

$$Pv = \sum_{E=o}^{N/4} C_N^E \, p^E \, (I-p)^{N-E} \left[\left(\frac{I}{2}\right)^N \sum_{e=E+I}^{N-E-I} C_N^e \right]^{M-I}$$

The various values of Pv as a function of N and p are plotted on the curve in Figure 2.

To obtain a probability Pv of $1-10^{-10}$ for a p value of 10^{-2}, a length of 65 bits is required.

$$\boxed{N = 65 \text{ bits} \longrightarrow Pv = I - 6{,}80\ IO^{-II}}$$

A length of 63 bits would only give $Pv = 1\text{-}1.92\ 10^{-10}$

2. Code Composition

A 65-bit code cannot be achieved using a PN code since:

$$2^6\text{-}I < 65 < 2^7\text{-}I = I27$$

- if there are no restraints on the ground due to correlator problems, the simplest solution would be to use a 127-bit PN code; however, this length is clearly excessive.

- if it is desirable to retain a length of 65 bits, then a search must be made for the best codes. This requires an examination of all possible configurations -- for 65 bits, the number is 2^{65}-1. This number can be reduced to about 1/4 by taking account of the mirror, complementary, and mirror complementary configurations of any given configuration. Allowance can be made for the fact that sequences formed by 13 and 5-bit configurations are repeated several times. This study requires an extensive computation time.

An interim solution would be to add two bits (or three, if two is not enough) to a 63-bit synchronization code. This might allow composition of a 65- or 66-bit code with a minimum of difficulty which would give satisfactory results.

IV. LINE CUT INTO SEVERAL BLOCKS /11

A single synchronizing signal per line allows true code detection with the probability required. But some consideration must also be given to maintaining lock-on. The problem of bit shift must be added to that of bit error.

In the event of bit shift at the primary synchronizer output, the cycle can be lengthened or shortened by several bits. From the point of view of the secondary synchronizer, this problem is easily resolved by opening a window on the theoretical position of the code, and allowing it to remain locked on the code if its position is slightly sifted. However, a bit shift renders the information useless, since it is impossible to know at which portion of the cycle bit shift occurred.

In order to reduce the loss of information due to bit shift, the line can be segmented. It would seem at first glance that this would even reduce the length of the synchronization word, but we will see that this is not true.

Segmenting the line also removes the inconvenience of a stationary information word from one cycle to the other. There is no longer any need, therefore, to use an alternating code.

Let Pv represent the true detection probability for cycle synchronization, and Pl the probability that the line is correctly synchronized. If the line is divided into b blocks:

Pl = Probability (that the b blocks are correctly synchronized)

$$\geqslant (Pv)^b$$

An upper limit for Pv is determined by writing that $Pl = (Pv)^b$.

Pv is developed by the formula on Page 7 where M = 70,000/b.

The following table gives various values for Pv as a function of N and of b for $p = 10^{-2}$.

N	I - Pl			
	b=I	b=5	b=IO	
55	6,I6703 IO^{-9}	6,I6852 IO^{-9}	6,I6872 IO^{-9}	
56	3,66420 IO^{-9}	3,66486 IO^{-9}	3,66495 IO^{-9}	
57	2,I7743 IO^{-9}	2,I7772 IO^{-9}	2,I7776 IO^{-9}	
58	I,294IO IO^{-9}	I,29423 IO^{-9}	I,29425 IO^{-9}	
59	7,69226 IO^{-IO}	7,69284 IO^{-IO}	7,6929I IO^{-IO}	
60	4,57297 IO^{-IO}	4,57322 IO^{-IO}	4,57326 IO^{-IO}	
6I	2,7I895 IO^{-IO}	2,7I906 IO^{-IO}	2,7I907 IO^{-IO}	
62	I,6I68I IO^{-IO}	I,6I686 IO^{-IO}	I,6I687 IO^{-IO}	
63	9,6I555 IO^{-II}	9,6I577 IO^{-II}	9,6I579 IO^{-II}	
64	5,7I929 IO^{-II}	5,7I939 IO^{-II}	5,7I940 IO^{-II}	
65	3,40222 IO^{-II}	3,40226 IO^{-II}	3,40227 IO^{-II}	
66	2,024II IO^{-II}	2,024I3 IO^{-II}	2,024I3 IO^{-II}	/12
67	I,204357 IO^{-II}	I,204365 IO^{-II}	I,2004366 IO^{-II}	
68	7,I6679 IO^{-I2}	7,I6682 IO^{-I2}	7,I6683 IO^{-I2}	
69	4,2652I IO^{-I2}	4,265230 IO^{-I2}	4,26523I IO^{-I2}	
70	2,538648 IO^{-I2}	2,538655 IO^{-I2}	2,538656 IO^{-I2}	

Pl varies very little as a function of b. As a result, even if the line is divided into several cycles it will be necessary to place a synchronization word (63 bits) at the beginning of each cycle.

V. CHOICE OF A SOLUTION /13

There are four possibilities:

- a nonalternating 63-bit PN code at the beginning of each line;
- a 65-bit alternating code at the beginning of each line. It would be necessary to conduct an investigation to determine its composition,
- an alternating, 127-bit, PN code,
- a segmented line with a nonalternating, PN code of 63 bits at the beginning of each cycle.

Depending on the importance of bit shift, it is desirable to divide the line. The more important the bit shift is, the more the line will have to be divided. The choice between a divided or nondivided line can only be made after measurements on various primary synchronizers have given usable results.

If it is necessary to divide the line: nonalternating, PN code of 63 bits, at the beginning of each cycle is used.

If a single synchronization word is used per line it is preferable to use an alternating code which compensates for the stationary character of the frame signal from one line to another.

It is equally desirable to use a PN code which it is possible to create easily and which does not require a long study with respect to its composition.

According to the hypotheses we have used, however, the use of the PN code requires an extension in length to 127 bits if it is desirable to alternate it. These hypotheses only take into account the search phase. Since the probability of remaining locked on one position is greater than the probability to lock-on in search phase, simplified calculations (taking account of the operation described in Annex 2) show that it is possible to reduce code length from 65 to 63 bits. These results are plotted on the curves in Figure 3.

If a single synchronization word per line is used, a PN code of 63 bits is utilized, and it is alternated from one line to the other.

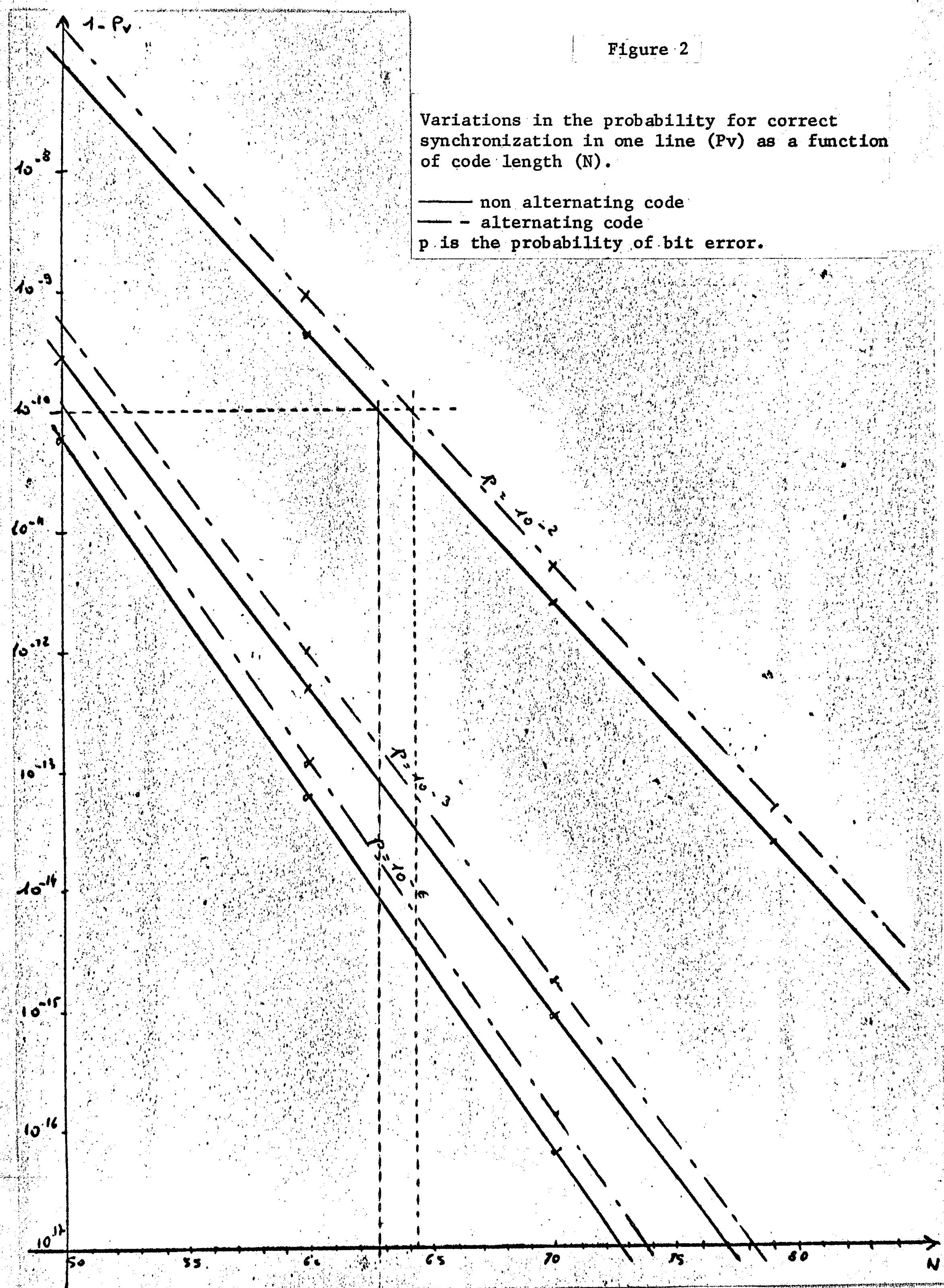
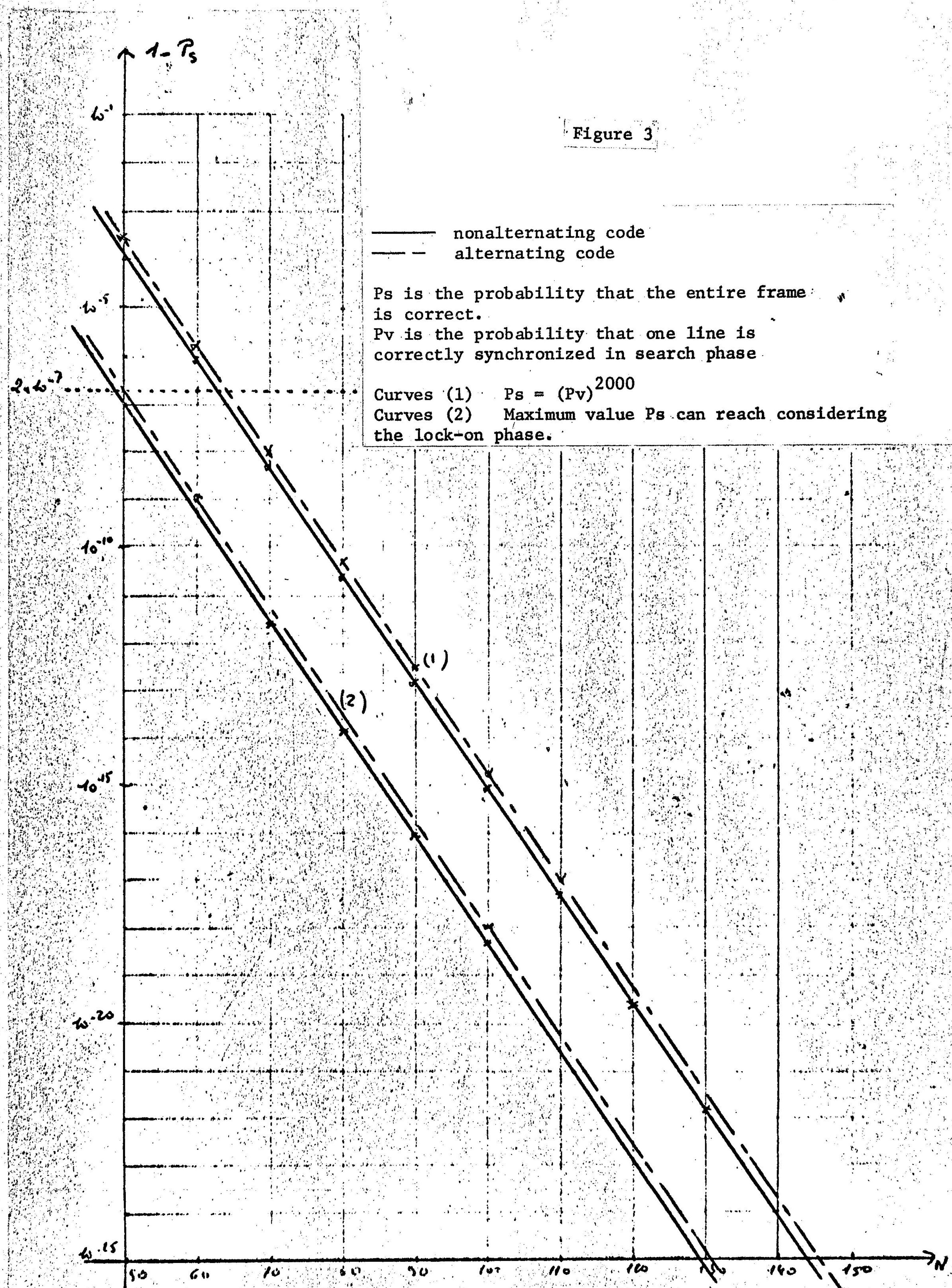


Figure 2

Variations in the probability for correct synchronization in one line (Pv) as a function of code length (N).

——— non alternating code
—— - alternating code
p is the probability of bit error.

Figure 3



———— nonalternating code
— — alternating code
Ps is the probability that the entire frame is correct.
Pv is the probability that one line is correctly synchronized in search phase
Curves (1) Ps = $(Pv)^{2000}$
Curves (2) Maximum value Ps can reach considering the lock-on phase.
1-Ps
N
(1)
(2)

APPENDIX I

SECONDARY SYNCHRONIZATION
SEARCH PHASE
FIXED THRESHOLD - VARIABLE THRESHOLD

We are dealing here with cycle synchronization, without word synchronization.

Review of Definitions

- Bit : primary information quantity, with value of 0 or 1,
- Word : coded value of a physical quantity of information; a word is made up of several bits,
- Cycle : a cycle is composed of several words. A cycle synchronization code marks the beginning of a cycle.
- Format : (or subcycle) : an assembly of cycles.

Cycle Synchronization

Marking the beginning of a cycle is done by a search for a pre-determined configuration of N consecutive bits placed in a precise location in the cycle (generally at the beginning).

This search is made by a correlator which compares the configuration of bits being sought (reference code) with the various configurations which appear successively in the message.

If M is the number of bits in a cycle, there are M different correlator positions in reference to the cycle; only one of these positions is the synchronization code position.

The correlator effects a bit to bit comparison between the N bits of the reference code and N bits of the message. It detects a certain number of errors (or mismatches) between these two configurations.

If the transmission were perfect, the correlator would detect no error in the code position. Probability of marking the code in correct position would then be equal to the probability that for any position whatever the bit configuration obtained would be different from that of the code. A value is obtained, then, equal to: /2

$$\left[1 - \left(\frac{1}{2}\right)^{N}\right]^{M-1}$$

In reality, the transmission has a bit error probability p, and detection of of the exact code position is based on the fact that in spite of bit errors more mismatches can be expected at random positions than at the code position.

In the following paragraphs, it is assumed that all bits other than the code are random and independent one from the other.

It is also assumed that the bit errors are random and independent.

Fixed Threshold Method

Synchronization code position search is carried out when the number of correlator output errors exceeds a pre-set threshold. The code is said to have been found when the number of errors is equal to or lower than the threshold.

If Pv is called the probability of true detection (that is, the probability of finding the code in its proper location), then Pv is the probability that for the entire preceding cycle the number of errors is higher than the threshold, while for the code the number of errors is equal to or lower than the threshold.

Let: N be the number of bits in the code
M be the total number of bits in the cycle
e be the number of errors in the correlator output
E be the error threshold
p be the bit error probability

- The probability that there are e errors in the code is the probability that e bits are incorrect while the others (N-e) are correct. The incorrect e bits can be placed in the code in n different ways, n being the number of combinations of N bits e to e

$$n = C_N^e$$

This probability is, then: $C_N^e \; p^e \; (I-p)^{N-e}$ /3

From that, it can be deduced that the probability for at least E errors in the code is:

$$\sum_{e=o}^{E} C_N^e \; p^e \; (I-p)^{N-e}$$

- The probability that there are e errors in any position whatever is the probability that there are e bits mismatched with the reference code and N-e bits matched.

As in the preceding case, mismatched e bits can occupy n different positions ($n = C_N^e$), and each bit has a 1/2 probability of being matched or of being mismatched.

This probabilty is, then: $C_N^e \left(\frac{I}{2}\right)^N$

The probability that there are more than E errors at any position whatever becomes:

$$\sum_{e=E+I}^{E} C_N^e \left(\frac{I}{2}\right)^N = I - \left(\frac{I}{2}\right)^N \sum_{e=o}^{E} C_N^e$$

- The probability of true detection Pv is written:

$$\boxed{Pv = \left[\sum_{e=o}^{E} C_N^e \; p^e \; (I-p)^{N-e}\right] x \left[I-\left(\frac{I}{2}\right)^N \sum_{e=o}^{E} C_N^e\right]^{M-I}}$$

One can also compute the probability (Pa) that there is no detection (the number of errors always exceeds the threshold).

$$Pa = \left[\sum_{e=E+I}^{N} C_N^e \; p^e \; (I-p)^{N-e}\right] x \left[I-\left(\frac{I}{2}\right)^N \sum_{e=o}^{E} C_N^e\right]^{M-I}$$

$$Pa = \left[I - \sum_{e=o}^{E} C_N^e \; p^e \; (I-p)^{N-e}\right] x \left[I-\left(\frac{I}{2}\right)^N \sum_{e=o}^{E} C_N^e\right]^{M-I}$$

- The probability Pf of false detection is expressed by:

$$\boxed{Pf = I - Pa - Pv}$$

$$Pf = I - \left[I - \left(\frac{I}{2}\right)^N \sum_{e=o}^{E} C_N^c \right]^{M-I} \qquad /4$$

The values of Pv, Pa, and Pf vary as a function of the threshold.

The optimum threshold would be the threshold value for which Pv is maximum.

Variable Threshold Method

For each correlator position there is a comparison between the number of errors obtained and a particular stored threshold.

If the number of errors is lower than the threshold, the value found becomes the new threshold, and the bit counter is reset to zero. Otherwise, the counter simply adds one unit.

As a result, the synchronization code will be detected if the number of errors found at its position is lower than the number of errors found at preceding positions. Confirmation will be received that the position found for the code is in effect the proper one if the bit counter reaches value M.

It is apparent that this method entails synchronization of the secondary synchronizer even when there is no message. For this reason, the maximum error threshold can be taken as N/4, an intermediate value between the mean number of errors when there is no message (N/2) and the number of errors which can be expected with an error probability of 10^{-1} (10).

Based on what has been stated above, the probability of detecting the synchronization code is written:

$$\boxed{Pv = \sum_{E=o}^{N/4} \left\{ C_N^E \quad p^E \; (I-p)^{N-E} \quad \left[I-(\tfrac{I}{2})^N \quad \sum_{e=o}^{E} \; C_N^e \right]^{M-I} \right\}}$$

There will be no detection if the number of errors always remains higher than N/4.

$$Pa = \sum_{e=N/4+I}^{E} C_N^e \quad p^e \; (I-p)^{N-e} \quad \left[I-(\tfrac{I}{2})^N \; \sum_{e=o}^{N/4} \; C_N^e \right]^{M-I}$$

The probability of false detection is expressed: /5

$$Pf = I - (Pv + Pa)$$

In practice, $Pa \neq o$ and $Pf \neq I - Pv$

Comparison of the Two Methods

The variable threshold method gives the best results (that is, the highest Pv value) whatever the cycle length, the bit error probability, or the code length.

The following table gives values of Pv as a function of N (code length) and of p (probability of bit error) for a cycle length M of 70,000 bits.

The value of Pv used for the fixed threshold method is the value which corresponds to the optimum threshold Eo for each N and p value.

	$p = IO^{-2}$			$p = IO^{-3}$			$p = IO^{-6}$		
	variable threshold	fixed threshold		variable threshold	fixed threshold		variable threshold	fixed threshold	
N	I-Pv	I-Pv	Eo	I-Pv	I-Pv	Eo	I-Pv	I-Pv	Eo
40	I,53 IO^{-5}	I,37 IO^{-3}	3	2,08 IO^{-7}	6,I9 IO^{-5}	2	6,38 IO^{-8}	2,6I IO^{-6}	I
50	8,34 IO^{-8}	I,58 IO^{-4}	5	3,3I IO^{-IO}	I,52 IO^{-6}	3	6,23 IO^{-II}	4,40 IO^{-9}	I
60	4,57 IO^{-IO}	5,83 IO^{-6}	6	5,40 IO^{-I3}	3,70 IO^{-8}	4	6,09 IO^{-I4}	I,II IO^{-IO}	2
70	2,54 IO^{-I2}	6,23 IO^{-7}	7	8,96 IO^{-I6}	8,99 IO^{-IO}	5	5,96 IO^{-I7}	2,02 IO^{-I3}	2
80	I,42 IO^{-I4}	2,40 IO^{-8}	9	I,50 IO^{-I8}	2,I9 IO^{-II}	6	5,83 IO^{-20}	4,95 IO^{-I5}	3
90	8,05 IO^{-I7}	2,38 IO^{-9}	IO	2,55 IO^{-2I}	5,32 IO^{-I3}	7	5,70 IO^{-23}	9,43 IO^{-I8}	3
IOO	4,58 IO^{-I9}	9,85 IO^{-II}	I2	4,28 IO^{-24}	I,30 IO^{-I4}	8			

APPENDIX II

STUDY OF VARIOUS TYPES OF SECONDARY SYNCHRONIZER OPERATION /1

I OPERATION OF A STANDARD SECONDARY SYNCHRONIZER (Variable Threshold)

The operation of a secondary synchronizer is divided into three phases:

- search phase
- inspection phase
- lock-on phase

1 Search Phase

The two possible methods of search (fixed threshold and variable threshold) were described in Appendix I.

When the correlator finds a position with a high probability of being the code position (in a variable threshold, when the bit counter reaches value M -- the number of bits in the cycle), the secondary synchronizer leaves the search phase and begins the inspection phase.

2. Inspection Phase

The inspection phase serves to verify that detection is correct, that is, that the code position found is its actual position.

In the case of a variable threshold, when the bit counter reaches the value of M it is automatically reset to zero, and the number of errors found in the theoretical code position replaces the former threshold value. Confirmation is received if the number of errors remains higher than the new threshold in the M-1 position which follow in the cycle (that is, up until the counter reaches the new value of M). An invalidation is considered as received in the contrary case; that is, if there is at least one position for which the number of errors is below the threshold.

After a certain number of confirmations have been received successively, the system enters lock-on mode. /2

When a certain number of successive invalidations are received, the system returns to search.

3. Lock-on Phase

In lock-on phase, the system operates on the same principle as in inspection phase, with reactualization of the threshold at each theoretical code position.

The difference between the two phases lies in the presence of a divider which only registers an invalidation if the number of errors found at any one position is lower than E/k (E being the threshold value).

2. OPERATION IN THE METEOSAT SYSTEM

For METEOSAT, the quantity of information lost must be minimized when there is poor lock-on or when unlock occurs.

In the standard operation described above, it is necessary to wait for several cycles during inspection phase before deciding whether or not detection was correct. As a result, several cycles are incorrect if detection is incorrect; for METEOSAT that is represented by several lines if a single synchronization signal is used per line. If an incorrect detection has been made, this must be known quickly. This can be accomplished in the following manner:

Search phase is carried out in the standard way.

Inspection phase is limited to one cycle — that is, as soon as the bit counter reaches value M, the system switches to lock-on phase.

In lock-on phase, there is a comparison between the number E of errors obtained in the theoretical code position and the various values e obtained at each following position in the cycle. If $e < E/k$, (k being a number set in advance) for any position whatever, the system unlocks and returns to search beginning with that position. When a k value is large enough, one can consider there to be an unlock only when synchronization is actually lost.

APPENDIX 3

COMPUTING COEFFICIENTS Hm(E)

1. DEFINITION

All positions in the cycle do not contain only random bits. If N is the length of the code, there are 2N-2 positions for which the N bits compared by the correlator to the reference code are composed of m bits (the code bits) and N-m bits (random bits of the message). Degree of overlay is expressed by m. /1

For the code 00011101101, as an example, and with an overlay of seven, 1101101XXXX will be in the correlator (the X's representing random bits).

Each m value corresponds to two symmetrical positions (an overlay to the right and an overlay to the left) which have the same characteristics.

Hm(E) is used to designate the probability that for overlay m, the number of errors at the correlator output will be equal to or lower than E.

Computing coefficients Hm(E) brings into play a given code's truncated correlation properties. (By "truncated correlation" we mean correlation between bits in the overlay.)

Let c represent the number of mismatched bits.

Let a represent the number of matched bits (a = m-c).

In the code 00011101101 with an overlay of seven, for example, c = 4 and a = 3.

```
reference code ce         | 0 0 0 I I I 0 | I I 0 I |
message       X X 0 0 0 I | I I 0 I I 0 I | X X X X | X X
                          |<-- overlay -->|
                          |<------ correlator ----->|
                                   position
```

2. COMPUTING COEFFICIENTS Hm(E)

If there are e errors at the correlator output for an m overlay, these e errors are divided into:

α errors for the m code bits
β errors for the random bits (b = N−m)

$$e = \alpha + \beta$$

$$\alpha \leq m \qquad \beta \leq b$$

$$Hm(E) = \sum_{e=0}^{E} \text{Probability (e errors at the correlator output)}$$

$$Hm(E) = \sum_{e=0}^{E} \; \sum_{\alpha=0}^{Min(e,m)} \text{Probability } (\alpha \text{ errors in the code bits}) \times \text{Probability } (\beta \text{ errors in the random bits})$$

$$Hm(E) = \sum_{\alpha=0}^{Min(E,m)} \text{Probability } (\alpha \text{ errors in the code bits}) \sum_{\beta=0}^{Min(E-\alpha,b)} \text{Probability } (\beta \text{ errors on the random bits})$$

Min (i,j) designates the smallest values of i and j.

$$\text{Probability } (\beta \text{ errors in the b random bits}) = \left(\frac{1}{2}\right)^{b} C_{b}^{\beta}$$

Among the m code bits (without bit error) there are c mismatched bits which generate c errors in the correlator. When there are bits errors, the number of errors in the correlator is expressed as shown below (where i bits are incorrect among the mismatched bits and j bits are incorrect among the matched bits):

$$\alpha = c - i + j$$

The possible values of i and j depend on the respective values of c and α.

The minimum value of j corresponds to the minimum value of i, and the maximum value of j corresponds to the maximum value of i.

If $\alpha > c$ the minimum value of i is o which corresponds to $j = \alpha - c$

If $\alpha < c$ the minimum value of j is o, which corresponds to $i = c = \alpha$

In addition, since $i \leq c$, $j \leq \alpha$. And, since j is the number of incorrect /3
bits among the mismatched bits $j \leq m - c$

$$\begin{cases} j \leq \alpha \\ j \leq m - c \end{cases}$$

Consequently:

If $m-c > \alpha$, the maximum value of j is α, whhich corresponds to $i = c$.

If $m-c \leq$, the maximum value of j is $m-c$, which corresponds to $i = m-\alpha$

It can be inferred therefrom that:

$$Hm(E) = \sum_{\alpha=0}^{Min(E,m)} \sum_{i=Max(0,c-\alpha)}^{Min(c,m-\alpha)} C_c^i \, p^i (I-p)^{c-i} C_{m-c}^j \, p^j (I-p)^{m-c-j} \sum_{\beta=0}^{Min(E-\alpha,b)} \left(\frac{1}{2}\right)^b C_b^\beta$$

with $j = \alpha-c+i$ and $b = N-m$

Maximum (μ, ν) designates the largest values of μ and v.

Hm(E) can be written:

$$\boxed{H_m(E) = \left(\frac{1}{2}\right)^b \sum_{i=\mathrm{Max}(0,\,c-E)}^{c} C_c^i \, p^i (1-p)^{c-i} \sum_{j=0}^{\mathrm{Min}(m-c,\,E-c+i)} C_{m-c}^j \, p^j (1-p)^{m-c-j} \sum_{\beta=0}^{\mathrm{Min}(E-c+i-j,\,b)} C_b^\beta}$$

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